Depolarizing Resonances

- Vertical guide field strongest ⇒ axis of spin precession usually vertical.

  γG = "spin tune"
  (number of 360° precessions turn)

- Horizontal field components always present in focusing elements due to vertical betatron oscillations, ⇒ spectrum of horizontal field contains beat frequencies between field spectrum and vertical tune:

$$b_x(\theta) \propto \sum_n (\alpha_n \cos n\Theta + \beta_n \sin n\Theta)\cdot\cos\nu_z\Theta$$

$$\propto \sum_n [\alpha_n \cos(n\pm\nu_z)\Theta + \beta_n \sin(n\pm\nu_z)\Theta].$$

- **When γG equals one of these frequencies, coherent addition of effect of horizontal fields occurs, ⇒ resonance when**

  $\gamma G = n \pm \nu_z$ **(intrinsic resonance).**

- **In addition, there are horizontal fields due to orbit excursions leading to resonances when**

  $\gamma G = n$ **(imperfection resonance).**

Crossing a Depolarizing Resonance

- Spin precesses about non rotating axis in coordinate system rotating with resonant frequency.

- As a resonance is crossed, precession axis rotates from vertical to horizontal (on resonance) to vertical.

- Three cases to consider:

  → Slow crossing (many turns):
  Spin follows precession axis ⇒ spin flip.

  → Fast crossing (less than one turn):
  Spin will not be affected ⇒ no depolarization.

  → **Intermediate crossing speed (few turns):**
  Spin motion greatly disturbed
  ⇒ depolarization.

- **The Froissart–Stora formula gives an analytic expression for asymptotic crossing of an isolated resonance:**

$$\frac{P}{P_0} = 2e^{-\left[\frac{\pi \epsilon^2}{2(\alpha - d\gamma/d\Theta)}\right]} - 1.$$

Siberian Snakes

- 180° spin flip about a horizontal axis.

  longitudinal axis: 1. kind
  radial axis: 2. kind

- Undo on the second turn
  what you did in the first turn.

- **Spin precession axis always in the horizontal plane(1 snake).**

- **Two snakes on opposite sides with orthogonal precession axes:**
  ⇒ Spin precession axis vertical.

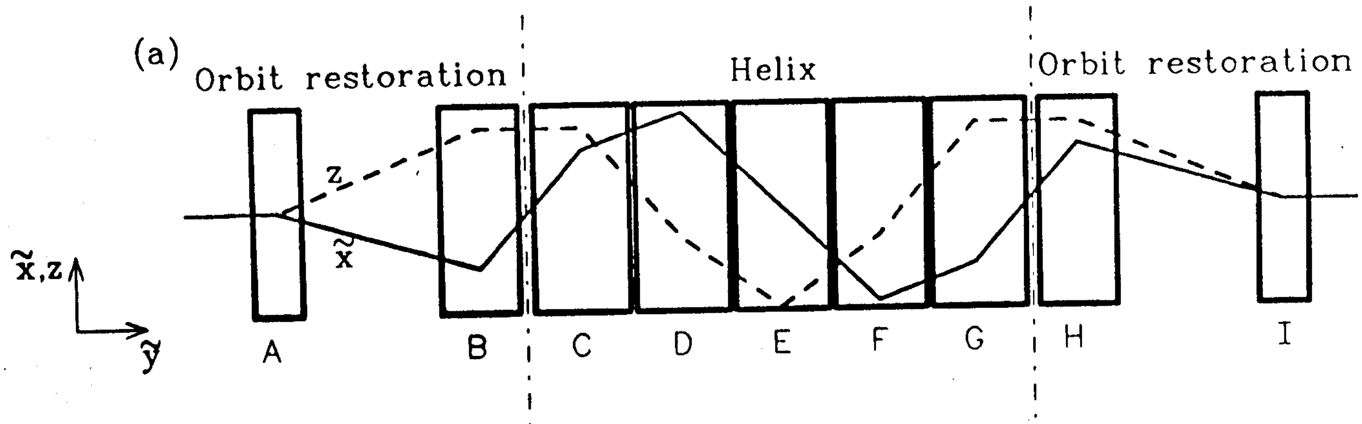
- **Siberian Snakes for high−energy applications are designed using bending magnets:**

  $\Theta_s = \gamma G \Theta_b$ for transverse fields;
  $\Theta_s = (1+G)\Theta_b$ for solenoidal fields. $\Theta_b = \frac{B\ell}{p} \cdot 0.3$

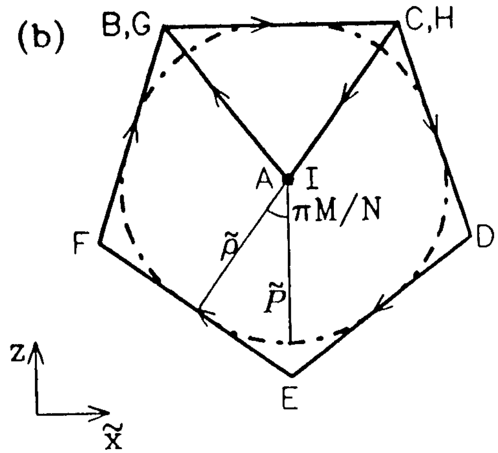
Examples of Siberian Snakes of 1. kind:

- Steffen Snake;
- Courant helical snake;
- **Helical snake with discrete magnets.**

Helical snake approximated with dipoles



side view
(top ")



view along axis of helix

Helical snake - features

- Orbit excursions

$$\rho \approx \frac{L\sqrt{4M+1}}{4\pi M^2 \gamma G}$$ M = no. of twists

- Field integral

$$BL \approx \pi \sqrt{4M+1}\ \frac{m_p c}{e}$$

- Thus ρ decreases with $1/M$ while BL increases only with $\sqrt{M}$

- Dependence of parameters on no. of dipoles/twist is small.

4 dipoles/twist gives 2 orthogonal planes

Main Injector

2- fold symmetry ⇒ lots of resonances

⇒ use snake(s)

DEPOL run ⇒ $\hat{\varepsilon}_{intr.} \approx 0.2$ up to 180 GeV (for $\varepsilon^* = 10\pi \cdot 10^{-6}$ rad m)

⇒ one snake would be sufficient

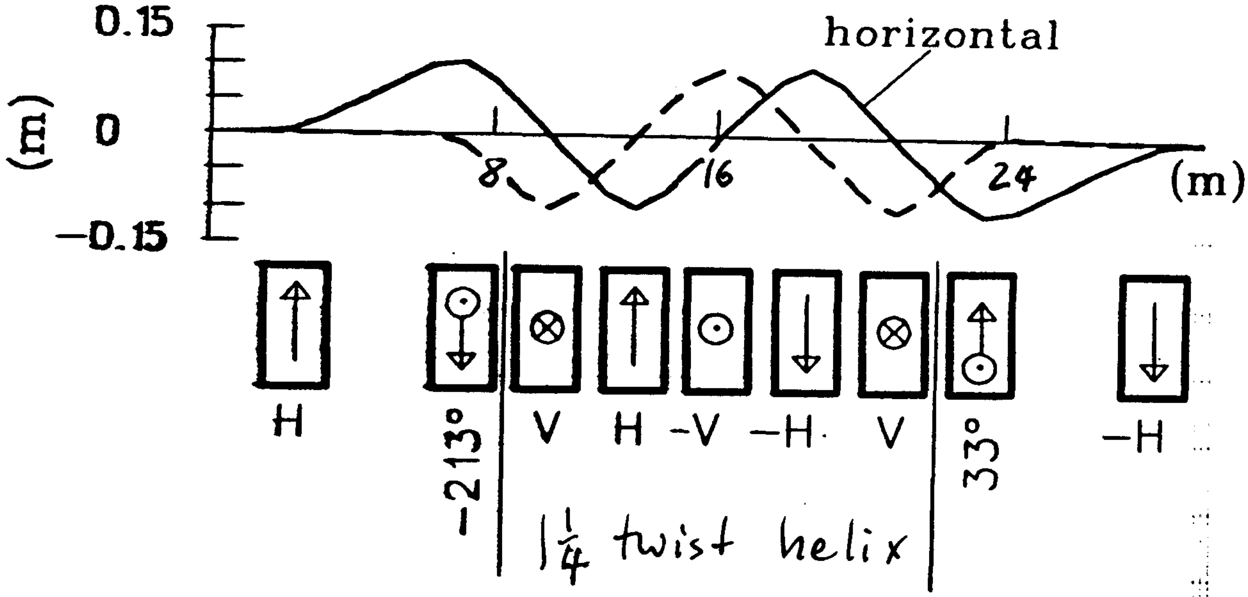
Advantage : methods to design snakes of 1st kind with small orbit exc. exist.

Dis adv. : may need tight tolerances on vertical orbit to control imperf. reso. sh

need to rotate spin in transfer line from Booster.

$1\frac{1}{4}$-twist helical snake

for M. I.



4 dipoles (5 in helix, 4 in orbit restoration)

At 8 GeV ($\gamma G \approx 17$)

5.5°/dipole in helix

1.4°/dipole outer O-r
5°/ " inner "

$B = 1.6\ T \Rightarrow L_{h.d.} = 2\ m$

$\int_{snake} B\,dl \approx 22\ Tm$, $L_{snake} \approx 28\ m$

Orbit exc. $\approx$ 10 cm

TRIUMF KF Driver m-e straight, with 2¼ twist scale, @ 3GeV