MI-0037
Eddy Current Induced Multipoles in the Main Injector

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1 Introduction

In a fast ramping machine such as the Main Injector, the eddy currents induced in the vacuum chamber walls by the time variation of the dipole magnet field have a non-negligible effect on field quality. As illustrated in figure 1, the problem can be considered as two dimensional; if the driving time varying field is vertical, the induced currents are longitudinal. Since the eddy currents are themselves time-varying, they induce additional eddy currents. In general, the eddy current distribution must be determined by solving the following equation [1]

$$\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J} - \sigma \frac{\partial \mathbf{A}}{\partial t} \tag{1}$$

where $\nu = 1/\mu$ is the reluctivity, σ is the electric conductivity and $\mathbf{J}$ is the imposed source density (e.g. the current circulating in the magnet windings). The term $-\sigma \frac{\partial \mathbf{A}}{\partial t}$ obviously represents the eddy currents. In practice, Equation (1) cannot be solved analytically and one must resort to a numerical solution. The code PE2D for example, solves (1) using finite element discretization in space and finite difference in time. Unfortunately, solving (1) numerically is expensive; furthermore, PE2D has limitations which we do not wish to discuss her.

In practice, the vacuum chamber is thin and the material which it is made of (stainless steel) possesses a relatively high resistivity. In such a situation, the "self-induced" eddy currents are negligible and it is sufficient to compute only the eddy currents due to the driving field. An a posteriori check of the validity of this approach is provided by the fact that the the field due to the eddy currents should be much smaller than the driving field.

[1] Equation (1) holds if displacement currents are neglected. This amounts to neglecting radiation.

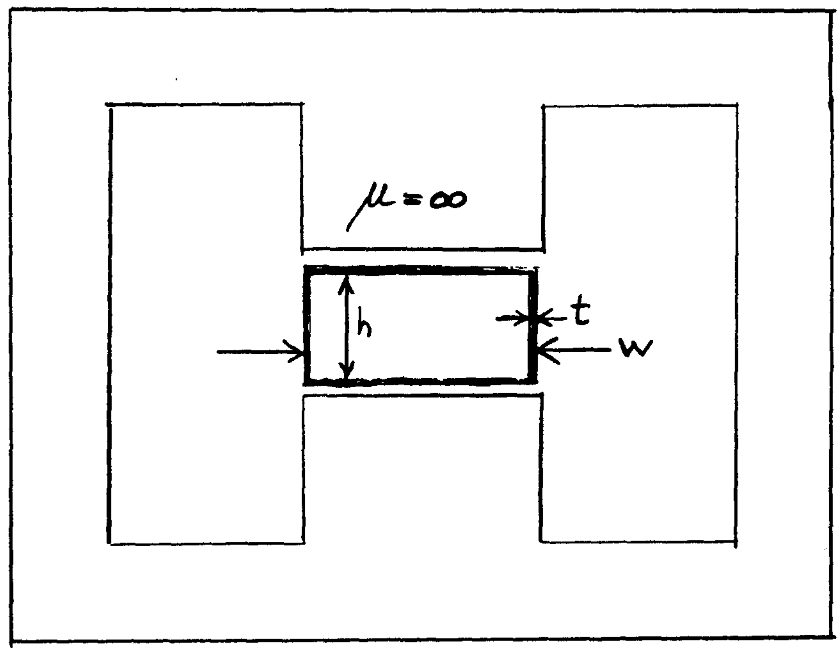

Figure 1: Problem geometry.

2 Simplified Theory

The eddy currents induced by a known driving field $\mathbf{B}_0$ can be calculated using Maxwell's equations for the curl and divergence of $\mathbf{E}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{E} = 0 \tag{3}$$

For the simple geometry illustrated in figure 1, the divergence equation is trivially satisfied and the curl equation becomes

$$\frac{dE_z}{dx} = \frac{dB_{0y}}{dt} \tag{4}$$

i.e.

$$E_{ez}(x) = \int \dot{B}_{0y}\, dx \tag{5}$$

$$= \dot{B}_{0y} x \tag{6}$$

and

$$J_e = \sigma \dot{B}_{0y} x \tag{7}$$

since by symmetry, the induced curent must be zero at $x = 0$. It is easily verified that the symmetry forces all the skew multipoles and the normal multipoles of order $2(2n + 1)$ to vanish. In such a situation, all the information about the magnetic field inside the vacuum chamber is contained in the behavior of B_{ey} in the midplane. Assuming that the magnet core is infinitely permeable and that the field lines are vertical across the gap the following expression is obtained for $B_{ey}(x, 0)$ by a straightforward application of Ampere's law [1].

$$B_{ey}(x) = \mu_0 \frac{\sigma \dot{B}_{0y} t}{g}(x^2 - \frac{w^2}{4} - \frac{w}{2} g) \quad \text{MKS} \tag{8}$$

where σ is the conductivity, $\dot{B}_0$ is the rate of variation of the main dipole field and x, w, g are as shown in figure 1. Separating the dipole and sextupole components yields

$$b_{e0} = -\mu_0 \frac{\sigma \dot{B}_{0y} t}{g}(\frac{w^2}{4} + \frac{w}{2} g) \tag{9}$$

$$b_{e2} = +\mu_0 \frac{\sigma \dot{B}_{0y} t}{g} \tag{10}$$

In these expressions, the signs of the multipoles are consistent with a **positive** main dipole field B_{0y} and a positive time derivative $\dot{B}_{0y}$. The dipole component of the induced field comes exclusively from the current flowing in the two vertical sides of the beam pipe. To the extent that these two sides form a loop, the induced dipole must, in accordance with Lentz's law, oppose any increase in the flux through that loop. As a result, the sign of the eddy induced dipole must be opposite to that of the the driving. The induced sextupole results from the current flowing in the horizontal walls of the chamber. These walls do not form a loop linking the driving field; therefore, it is not possible to use a simple argument to predict the sign of the sextupole. With

$$\begin{array}{lcll} B_{0y} & = & 0.205 & \mathrm{T} \\ \dot{B}_{0y} & = & 1.725 & \mathrm{T/s} \\ \sigma & = & 2.13 \times 10^{6} & \Omega^{-1} - \mathrm{m}^{-1} \\ t & = & 0.0016 & \mathrm{m} \\ g & = & 0.05 & \mathrm{m} \\ w & = & 0.10 & \mathrm{m} \\ h & = & 0.05 & \mathrm{m} \end{array}$$

one obtains, in normalized units (1 inch = 2.54 cm)

$$\begin{array}{lcl} b_{e1} & \simeq & -36 \\ b_{e2} & \simeq & +4.7 \end{array}$$

3 Analytical Solution

Expressions (9) and (10) have the merit of being explicit in w and g. However, the simplified analysis on which they are based does not provide any information about the higher order multipoles. The latter are non-zero because of the finite width of the beam pipe. More accurate results can be obtained by obtaining an a analytic solution for the eddy current induced field and expanding the latter in multipoles. This is the approach used by S.Y. Lee [2]. Assuming that the chamber is in between two infinitely permeable horizontal boundaries, it easily shown using the method of images that the complex field $B_y + iB_x$ at a point $z = x + iy$ produced by a filament at z_c is

$$B = \mu_0 \frac{I}{4g} \left[\tanh \frac{\pi(z - z_c^*)}{2g} + \coth \frac{\pi(z - z_c)}{2g} \right] \tag{11}$$

This field can be expanded about the origin

$$\begin{aligned} B &= \mu_0 \frac{I}{2g} \sum_{n=0}^{\infty} (\alpha_n + \beta_n) \left(\frac{\pi}{2g} \right)^n z^n && (12) \\ &= \sum_{n=0}^{\infty} B_n + iA_n && (13) \end{aligned}$$

where we have defined

$$\alpha_n = \left.\frac{\partial^n \tanh\left(z - \frac{\pi z_c^*}{2g}\right)}{\partial z^n}\right|_{z=0} \tag{14}$$

$$\beta_n = \left.\frac{\partial^n \coth\left(z - \frac{\pi z_c}{2g}\right)}{\partial z^n}\right|_{z=0} \tag{15}$$

S.Y. Lee has tabulated α_n and β_n up to order 8. His results are reproduced here for the reader's convenience.

n	α_n $t \equiv \tanh\left(-\frac{\pi z_c^*}{2g}\right)$ $c \equiv \cosh\left(-\frac{\pi z_c^*}{2g}\right)$
0	t
1	c^{-2}
2	$-2tc^{-2}$
3	$-2c^{-4} + 4t^2c^{-2}$
4	$16tc^{-4} - 8t^3c^{-2}$
5	$16c^{-6} - 88t^2c^{-4} + 16t^4c^{-2}$
6	$-272tc^{-6} + 416t^3c^{-4} - 32t^5c^{-2}$
7	$-272c^{-8} + 2880t^2c^{-6} - 1824t^4c^{-4} + 64t^6c^{-2}$
8	$7936tc^{-8} - 24576t^3c^{-6} + 7680t^5c^{-4} - 128t^7c^{-2}$

n	β_n $\tilde{t} \equiv \coth\left(-\frac{\pi z_c}{2g}\right)$ $s \equiv \sinh\left(-\frac{\pi z_c}{2g}\right)$
0	$\tilde{t}$
1	$-s^{-2}$
2	$2\tilde{t}s^{-2}$
3	$-2s^{-4} - 4\tilde{t}^2s^{-2}$
4	$16\tilde{t}s^{-4} + 8\tilde{t}^3s^{-2}$
5	$-16s^{-6} - 88\tilde{t}^2s^{-4} - 16\tilde{t}^4s^{-2}$
6	$272\tilde{t}s^{-6} + 416\tilde{t}^3s^{-4} + 32\tilde{t}^5s^{-2}$
7	$-272s^{-8} - 2880\tilde{t}^2s^{-6} - 1824\tilde{t}^4s^{-4} - 64\tilde{t}^6s^{-2}$
8	$7936\tilde{t}s^{-8} + 24576\tilde{t}^3s^{-6} + 7680\tilde{t}^5s^{-4} + 128\tilde{t}^7s^{-2}$

The vacuum chamber can be regarded as a continuous distribution of filaments

$$B(z) = \frac{\mu_0}{4\pi}\int I(x,y)\left[\coth\frac{\pi(z - z_c)}{2g} + \tanh\frac{\pi(z - z_c^*)}{2g}\right] dxdy \tag{16}$$

The multipoles of the total field are simply the sum of the multipoles due to each filament

$$B_n + iA_n = \frac{\mu_0}{2\pi n!}\int I(x,y)(\alpha_n + \beta_n)\left(\frac{\pi}{2g}\right)^n dxdy \tag{17}$$

Assuming that the current is constant throughout the thickness t of the chamber equation (17) becomes, in normalized form:

$$b_n + ia_n = \frac{\mu_0 \sigma t \dot{B}_0}{2\pi n! B_0} \left(\frac{\pi}{2g}\right)^{n+1} \int x(\alpha_n + \beta_n)\, ds \tag{18}$$

where ds is a differential element of path length.

4 Computer Code

A code has been written to compute the eddy current multipoles using the approach described in the preceding section. In particular, we were interested in studying the behavior of the multipoles as a function of the width of the chamber. The results are presented in figures 2-5. Although the dipole is very well predicted by the simplified expressions, the sextupole is independent of the width of the chamber only to the extent that $w > 2h$. The decapole and the 14-pole decay rapidly as the width of the chamber increases.

ASSUMPTIONS:

$$\begin{aligned}
B_0 &= 0.205 && \mathrm{T} \\
\dot{B}_0 &= 1.725 && \mathrm{T/s} \\
\sigma &= 2.13 \times 10^6 && \Omega^{-1} - \mathrm{m}^{-1} \\
t &= 0.0016 && \mathrm{m} \\
g &= 0.0508 && \mathrm{m} \\
h &= 0.0450 && \mathrm{m}
\end{aligned}$$

width w	dipole	sextupole	decapole	14-pole
0.0300	−5.827	+0.9132	−1.0774	+0.7525
0.0325	−6.427	+1.1456	−1.1890	+0.6798
0.0350	−7.053	+1.3795	−1.2672	+0.5815
0.0400	−8.384	+1.8497	−1.3270	+0.3581
0.0450	−9.821	+2.2978	−1.2810	+0.1573
0.0600	−14.766	+3.3589	−0.8657	−0.1221
0.0700	−18.573	+3.8108	−0.5814	−0.1313
0.0800	−22.771	+4.1024	−0.3609	−0.1006
0.0900	−27.345	+4.2995	−0.2265	−0.0675
0.1000	−32.294	+4.4138	−0.1359	−0.0425
0.1100	−37.604	+4.4823	−0.0804	−0.0257
0.1200	−43.275	+4.5225	−0.0471	−0.0153
0.1300	−49.303	+4.5452	−0.0274	−0.0089
0.1400	−55.689	+4.5574	−0.0158	−0.0052
0.1500	−62.431	+4.5630	−0.0091	−0.0030

NOTE: ALL DIMENSIONS IN MKS UNITS. MULTIPOLES IN NORMALIZED UNITS $\times 10^4$ @ 2.54 cm = 1 inch.

References

[1] Stephen Holmes, *Eddy Current Effects in the Main Injector Beam Tube* FNAL Internal Report MI-0013

[2] S.Y. Lee, *A multipole Expansion for the Field of Vacuum Chamber Eddy Currents*
BNL Internal Report AD/AP/TN-12

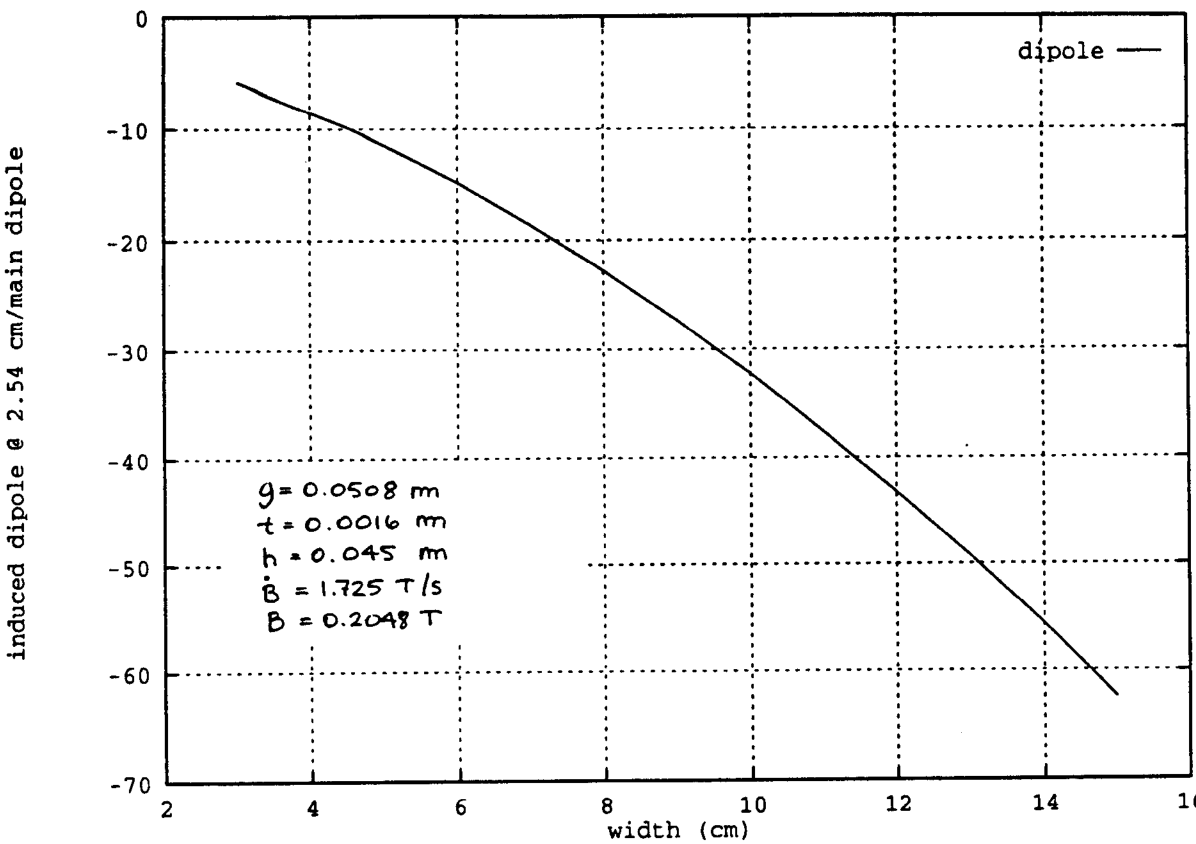

Figure 2: Variation of the induced dipole with the chamber width. $\sigma = 2.13 \times 10^6$.

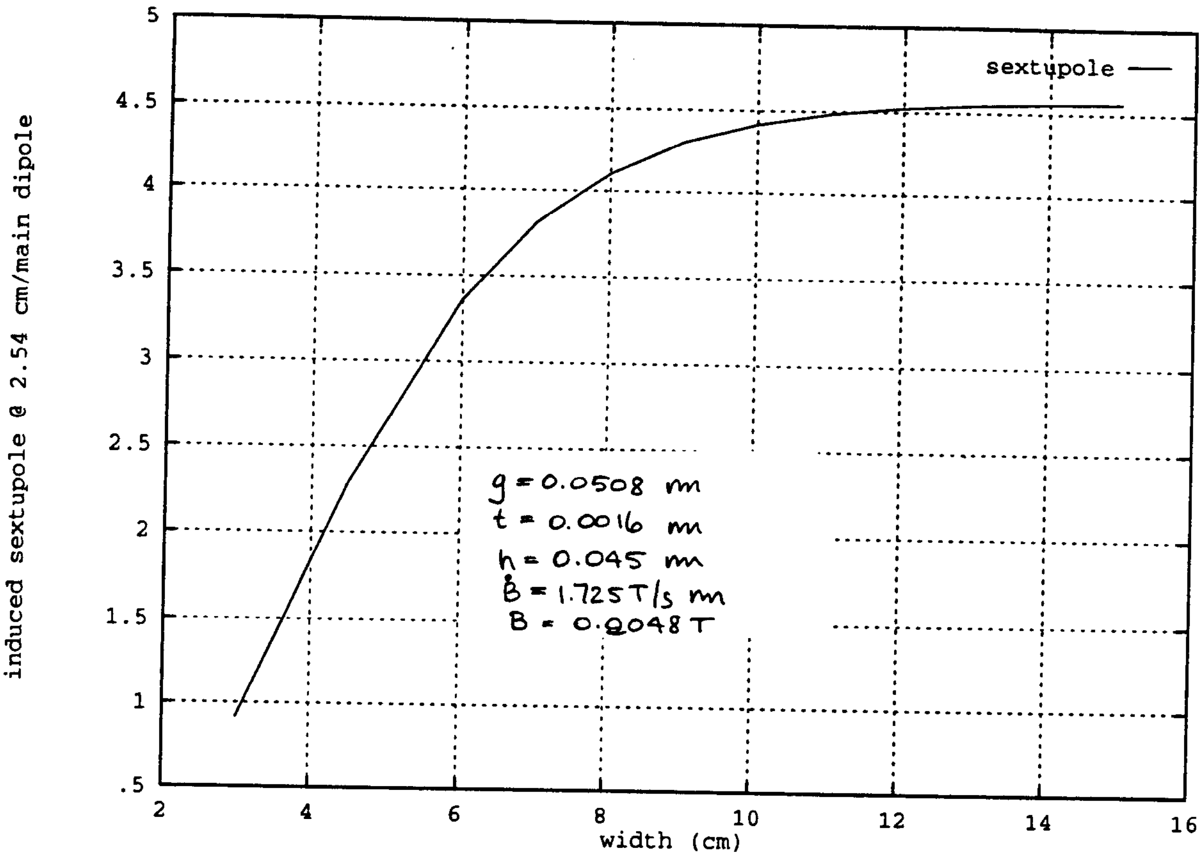

Figure 3: Variation of the induced sextupole with the chamber width. $\sigma = 2.13 \times 10^6$.

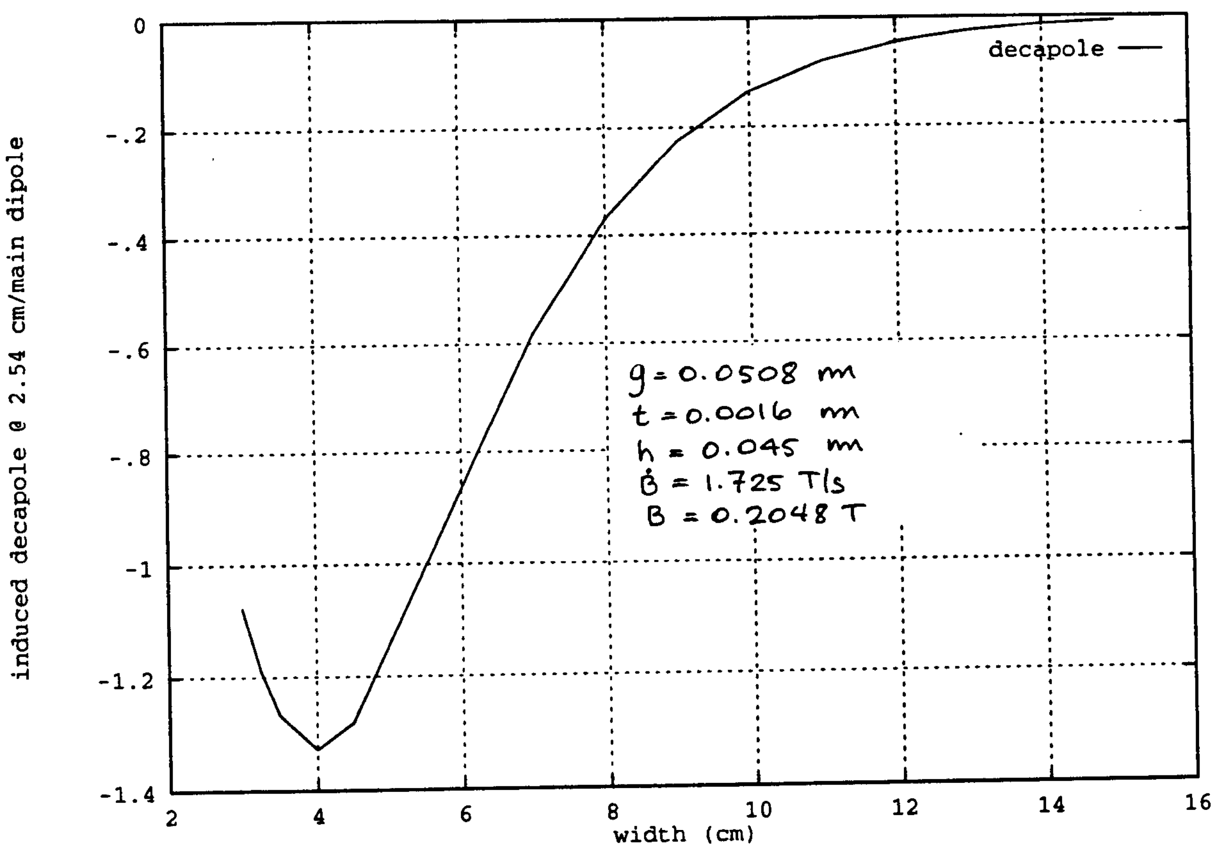

Figure 4: Variation of the induced decapole with the chamber width $\sigma = 2.13 \times 10^6$.